Całki Elementarne

- $\int \cos x dx = \sin x + C$
- $\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + C$
- $\int \frac{1}{\sin^2 x} dx = -\operatorname{ctg} x + C$
- $\int \frac{1}{1+x^2} dx = \operatorname{arctg} x + C$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$
- $\int \frac{1}{x} dx = \ln |x| + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\ln a} + C$
- $\int \sin x dx = -\cos x + C$

Często Używane Wzory

$$\int [f(x) \pm g(x)]\, dx = \int f(x)dx \pm \int g(x)dx$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int a \cdot f(x)dx = a \cdot \int f(x)dx$$

$$\int \frac{f'(x)}{2\sqrt{f(x)}} dx = \sqrt{f(x)} + C$$

$$\int f(ax+b)dx = \frac{1}{a}F(ax+b) + C$$

$$\int \frac{f'(x)}{(f(x))^n} dx = \frac{-1}{(n-1)\,(f(x))^{n-1}} + C$$

Ułamki Proste i Całki Wymierne

$$\frac{A}{(x+B)^n} \qquad A, B \in \mathbb{R} \quad n \in \mathbb{N}$$

$$\frac{Cx+D}{(x^2+bx+d)^n} \qquad b, d, C, D \in \mathbb{R} \quad n \in \mathbb{N} \quad \Delta = b^2 - 4d < 0$$

$$\int \frac{dx}{(1+x^2)^n} dx = \frac{x}{2(n-1)(1+x^2)^{n-1}} + \frac{2n-3}{2(n-1)} \int \frac{dx}{(1+x^2)^{n-1}} \qquad \text{dla } n \neq 1$$

Trygonometria

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$R(-\sin x, \cos x) = -R(\sin x, \cos x)$ podstawienie $\boxed{\cos x = t}$

$R(\sin x, -\cos x) = -R(\sin x, \cos x)$ podstawienie $\boxed{\sin x = t}$

$R(-\sin x, -\cos x) = R(\sin x, \cos x)$ podstawienie $\boxed{\operatorname{tg} x = t}$

W innych przypadkach podstawienie $\boxed{\operatorname{tg}\frac{x}{2} = t}$

$$\operatorname{tg} x = t \Rightarrow \sin x = \frac{t}{\sqrt{t^2+1}},\ \cos x = \frac{1}{\sqrt{t^2+1}}$$

$$\operatorname{tg}\frac{x}{2} = t \Rightarrow \sin x = \frac{2t}{1+t^2}\ \ \cos x = \frac{1-t^2}{1+t^2}$$

$$\sqrt{ax^2+bx+c} \quad \begin{array}{ccl} \to & \sqrt{x^2+k} & \text{dla } a>0 \\ \to & \sqrt{-x^2+k} & \text{dla } a<0 \end{array}$$

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2+k}}dx &= \ln|\sqrt{x^2+k}+x|+C \\
\int \sqrt{x^2+k}dx &= \frac{x}{2}\sqrt{x^2+k} + \frac{k}{2}\ln|\sqrt{x^2+k}+x|+C \\
\int \frac{x^2}{\sqrt{x^2+k}}dx &= \frac{x}{2}\sqrt{x^2+k} - \frac{k}{2}\ln|\sqrt{x^2+k}+x|+C \\
\int \frac{1}{\sqrt{-x^2+k}}dx &= \arcsin\frac{x}{\sqrt{k}}+C \\
\int \sqrt{-x^2+k}dx &= \frac{x}{2}\sqrt{-x^2+k} + \frac{k}{2}\arcsin\frac{x}{\sqrt{k}}+C \\
\int \frac{x^2}{\sqrt{-x^2+k}}dx &= -\frac{x}{2}\sqrt{-x^2+k} + \frac{k}{2}\arcsin\frac{x}{\sqrt{k}}+C
\end{aligned}$$

$$\int \frac{W_n(x)}{\sqrt{ax^2+bx+c}}dx = W_{n-1}(x)\sqrt{ax^2+bx+c} + A\int\frac{dx}{\sqrt{ax^2+bx+c}}$$