

## 1 Wideo 3

### 1.1 Zadanie 1

#### 1.1.1 a)

$$f(x) = 4x^3$$
$$f'(x) = 12x^2$$

#### 1.1.2 b)

$$f(x) = 3x + 1$$
$$f'(x) = 3$$

#### 1.1.3 c)

$$f(x) = -2x^2 + 2x - 1$$
$$f'(x) = -4x + 2$$

#### 1.1.4 d)

$$f(x) = -4x^7 + \frac{1}{5}x^5 - x$$
$$f'(x) = -28x^6 + x^4 - 1$$

### 1.2 Zadanie 2

#### 1.2.1 a)

$$f(x) = x^e$$
$$f'(x) = ex^{e-1}$$

#### 1.2.2 b)

$$f(x) = \frac{2}{x^2}$$
$$f'(x) = \frac{-4}{x^3}$$

#### 1.2.3 c)

$$f(x) = -3\sqrt[3]{x}$$
$$f'(x) = -x^{-\frac{2}{3}} = \frac{-1}{\sqrt[3]{x^2}}$$

1.2.4 d)

$$f(x) = \frac{2}{x\sqrt{x}}$$
$$f'(x) = \frac{-3}{x^2\sqrt{x}}$$

1.3 Zadanie 3

1.3.1 a)

$$f(x) = (x^2 - 2x)\frac{1}{x} = x - 2$$
$$f'(x) = 1$$

1.3.2 b)

$$f(x) = (x^3 + 2\sqrt{x})\sqrt[3]{x} = x^{\frac{10}{3}} + 2x^{\frac{5}{6}}$$
$$f'(x) = \frac{10}{3}x^{\frac{7}{3}} + \frac{5}{3}x^{-\frac{1}{6}} = \frac{10}{3}x^2\sqrt[3]{x} + \frac{5}{3\sqrt[6]{x}}$$

1.3.3 c)

$$f(x) = x \ln x$$
$$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

1.3.4 d)

$$f(x) = e^x \sin x$$
$$f'(x) = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$$

1.4 Zadanie 4

1.4.1 a)

$$f(x) = \frac{2x + 1}{x^2}$$
$$f'(x) = \frac{2 \cdot x^2 - (2x + 1) \cdot 2x}{x^4}$$

1.4.2 b)

$$f(x) = \frac{5x^2 + 2}{2x - 1}$$
$$f'(x) = \frac{10x(2x - 1) - 2(5x^2 + 2)}{(2x - 1)^2}$$

1.4.3 c)

$$f(x) = \frac{x^2 + 2x}{\sqrt{x^2 + 1}}$$
$$f'(x) = \frac{(2x + 2)\sqrt{x^2 + 1}}{x^2 + 1}$$

1.4.4 d)

$$f(x) = 2^x \cdot 4^{-x}$$
$$f'(x) = 2^x \ln 2 \cdot 4^{-x} + 2^x \cdot (-4^{-x} \ln 4)$$

1.5 Zadanie 5

1.5.1 a)

$$f(x) = (\sqrt{x} + 2)\left(\frac{1}{\sqrt{x}} - 1\right) = 1 - \sqrt{x} + 2x^{-\frac{1}{2}} - 2$$
$$f'(x) = -\frac{1}{2\sqrt{x}} - \frac{1}{x\sqrt{x}}$$

1.5.2 b)

$$f(x) = \frac{\arcsin x}{e^x}$$
$$f'(x) = \frac{\frac{1}{\sqrt{1-x^2}} \cdot e^x - \arcsin x \cdot e^x}{e^{2x}}$$

1.5.3 c)

$$f(x) = \frac{1}{3^x} = 3^{-x}$$
$$f'(x) = -3^{-x} \ln 3$$

**1.5.4 d)**

$$f(x) = \frac{(x^2 + 1)^2}{\ln x}$$
$$f'(x) = \frac{2(x^2 + 1) \cdot 2x \cdot \ln x - (x^2 + 1)^2 \frac{1}{x}}{\ln^2 x}$$

**1.6 Zadanie 6**

**1.6.1 a)**

np.  $f(x) = 4x$ ,  $f(x) = 4x + 22$ ,  $f(x) = 4x - \pi$

**1.6.2 b)**

np.  $f(x) = \frac{x^2}{2}$ ,  $f(x) = \frac{x^2}{2} + 81$

**1.6.3 c)**

np.  $f(x) = \frac{x^3}{3}$ ,  $f(x) = \frac{x^3}{3} - 7$

**1.6.4 d)**

np.  $f(x) = -\cos x$ ,  $f(x) = -\cos x - 44$

Ciekawostka - wyznaczanie funkcji, gdy znana jest jej pochodna nazywamy całkowaniem

**1.7 Zadanie 7**

**1.7.1 a)**

$$f(x) = 3 \cos x \cdot \arcsin x$$
$$f'(x) = -3 \sin x \cdot \arcsin x + 3 \cos x \cdot \frac{1}{\sqrt{1-x^2}}$$

**1.7.2 b)**

$$f(x) = \frac{\arccos x}{\sqrt{x}} \cdot e^x$$
$$f'(x) = \frac{\left(\frac{-1}{\sqrt{1-x^2}}e^x + \arccos x \cdot e^x\right)\sqrt{x} - \arccos x \cdot e^x \frac{1}{2\sqrt{x}}}{x}$$

1.7.3 c)

$$f(x) = \sqrt{x} \frac{\operatorname{ctg} x}{e^x} \operatorname{arctg} x$$

$$f'(x) = \frac{\left[ \frac{1}{2\sqrt{x}} \operatorname{ctg} x \cdot \operatorname{arctg} x + \sqrt{x} \left( \frac{-1}{\sin^2 x} \operatorname{arctg} x + \operatorname{ctg} x \cdot \frac{1}{1+x^2} \right) \right] e^x - \sqrt{x} \operatorname{ctg} x \operatorname{arctg} x \cdot e^x}{e^{2x}}$$

1.7.4 d)

$$f(x) = \ln x \cdot \log_4 x \cdot \operatorname{tg} x$$
$$f'(x) = \frac{1}{x} \cdot \log_4 x \cdot \operatorname{tg} x + \ln x \left( \frac{1}{x \ln 4} \operatorname{tg} x + \log_4 x \frac{1}{\cos^2 x} \right)$$