1 Wideo 1

1.1 zadanie 1

a)

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{h \to 0} \frac{-(x_0 + h)^2 + x_0^2}{h}$$

$$f'(-1) = \lim_{h \to 0} \frac{-(-1 + h)^2 + (-1)^2}{h} = \lim_{h \to 0} \frac{-h^2 + 2h - 1 + 1}{h} = \lim_{h \to 0}(-h+2) = 2$$

b)

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{h \to 0} \frac{-(x_0 + h)^2 + x_0^2}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{-(1 + 2h + h^2) + 1}{h} = \lim_{h \to 0} -(2 + h) = -2$$

c)

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{h \to 0} \frac{(x_0 + h)^3 - x_0^3}{h}$$

$$f'(0) = \lim_{h \to 0} \frac{(0 + h)^3 + 0^2}{h} = \lim_{h \to 0} \frac{h^3}{h} = \lim_{h \to 0} h^2 = 0$$

d)

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{h \to 0} \frac{(x_0 + h)^3 - x_0^3}{h}$$

$$f'(2) = \lim_{h \to 0} \frac{(2 + h)^3 + 2^2}{h} = \lim_{h \to 0} \frac{8 + 3 \cdot 2^2 h + 3 \cdot 2h^2 + h^3 - 8}{h} =$$

$$= \lim_{h \to 0} \frac{12h + 6h^2 + h^3}{h} = \lim_{h \to 0}(12 + 6h + h^2) = 12$$

e)

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{h \to 0} \frac{\frac{4}{x_0+h} - \frac{4}{x_0}}{h}$$

$$f'(-2) = \lim_{h \to 0} \frac{\frac{4}{-2+h} - \frac{4}{-2}}{h} = \lim_{h \to 0} \left(\frac{4}{h(h-2)} + \frac{2}{h} \right) =$$

$$= \lim_{h \to 0} \left(\frac{4}{h(h-2)} + \frac{2(h-2)}{h(h-2)} \right) = \lim_{h \to 0} \frac{2h}{h(h-2)} = -1$$

f)
g)
h)
i)

j)

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} \frac{\sqrt{1-2(x_0+h)}-\sqrt{1-2x_0}}{h}$$

$$f'(0) = \lim_{h\to 0} \frac{\sqrt{1-2h}-\sqrt{1}}{h} = \lim_{h\to 0} \frac{(\sqrt{1-2h}-1)(\sqrt{1-2h}+1)}{h(\sqrt{1-2h}+1)} =$$
$$= \lim_{h\to 0} \frac{1-2h-1}{h(\sqrt{1-2h}+1)} = -1$$

k*)

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} \frac{\cos(7(x_0+h))+3-(\cos(7x_0)+3)}{h}$$

$$f'(1) = \lim_{h\to 0} \frac{\cos(7(1+h))+3-(\cos(7)+3)}{h} = \lim_{h\to 0} \frac{-2\sin\frac{14+7h}{2}\sin\frac{7h}{2}}{h} =$$
$$= \lim_{h\to 0} \frac{-2\sin\frac{14+7h}{2}\sin\frac{7h}{2}}{\frac{2}{7}\cdot\frac{7h}{2}} = \frac{-2\sin\frac{14}{2}}{\frac{2}{7}} = -7\sin 7$$

skorzystano w tym punkcie ze wzorów

$$\lim_{h\to 0} \frac{\sin(ah)}{ah} = 1 \tag{1}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Dowód 1 opierający się na twierdzeniu o 3 funkcjach i geometrii płaskiej można znaleźć chociażby w Stefan Banach „Rachunek różniczkowy i całkowy” Warszawa 1957, PWN